23. MATHEMATICS

DETAILS OF SYLLABUS

UNIT I - BASIC MATH

1. ALGEBRA

Basic properties of real and complex numbers. Absolute value. Polar form of a complex number. De Moivre's Theorem and complex n^{th} roots.
Polynomials and polynomial equations, remainder and factor theorems. Quadratic equations. Systems of linear equations and their consistency. Matrix methods of checking consistency and solving systems of linear equations. Algebra of matrices. Basic combinatorics involving permutations and combinations.

2. ANALYTIC GEOMETRY

Coordinates of points in plane or space. Distance in terms of coordinates. Coordinates of points on line determined by two specified points. Slope of a line.
Representation of curves in a plane as equations and vice versa, especially straight lines, circles and conics. Inferring geometric properties of plane curves from the algebraic properties of their equations and vice versa.
Direction cosines and direction ratios of a line in space. Equations of lines in space. Coplanar and non-coplanar lines. Equations of planes and spheres.

3. SETS AND FUNCTIONS

General ideas of sets, especially sets of numbers and operations on sets. Real valued functions as transformations, domain and range of such functions. Injective and surjective functions, bijections and inverses. Graphs of real valued functions, especially polynomials and trigonometric functions.
Composition of functions

4. CALCULUS

Intuitive idea of limits of functions, differentiability, derivative as slope. Derivatives of polynomial functions, exponential function and trigonometric functions; derivatives of sums and products, composite and inverse functions. Increasing and decreasing functions, local extrema, simple applications.

Integration as anti-differentiation, integral as sum. Areas under curves and volumes of solids of revolution, using integration.

UNIT II - REAL AND COMPLEX ANALYSIS

Limits

Convergence sequences and series of real and complex numbers. Geometric and harmonic series.
Sequences and series of real and complex functions, point-wise and uniform convergence. Power series, radius of convergence. The exponential series. Limits and continuity of real and complex functions

Differentiation

Differentiation of real and complex functions. Analytic functions. Power series as analytic functions, extension of exponential and trigonometric functions to complex numbers. Branches of logarithm. Isolated singularities of complex function.

Integration

Riemann integrals and Riemann Stieltjes integrals of real valued functions. The concepts of Lebesgue measure and Lebesgue integral of real valued functions. Line integrals of complex valued functions, Cauchy’s Theorem and Integral Formula for complex functions. Contour integration.

Analytic functions

Properties of complex analytic functions, such as infinite differentiability, power series expansion, isolated zeros. Liouvilles’ Theorem. Open Mapping Theorem. Maximum Modulus Theorem.
Cauchy-Riemann Equations, harmonic conjugates. Conformal mappings, Mobius transformations.

UNIT III - ABSTRACT ALGEBRA

Rings

Ring of integers and ring of polynomials over real numbers. Integers modulo n. Finite rings.
Commutative and non-commutative rings. Ideals, maximal ideals, prime ideals. Quotients. Homomorphisms and isomorphisms. Homomorphic images as quotients.
Divisors of zeros. Integral domains. Euclidean domains. Factorization, units, associates, primes.
Primitive polynomials.

Fields

Rational numbers, real numbers and complex numbers as fields. Integers modulo a prime number. Finite fields. Finite integral domains are fields. Polynomials over fields, reducibility and irreducibility. Algebraic and transcendental extensions of fields. Splitting fields.

Vector spaces

Vector spaces over a field, especially over real numbers and complex numbers. Linear independence and dependence. Basis. Dimension. Linear subspaces and quotients. Geometry of R^2 and R^3
Linear maps. Representation of linear maps between finite dimensional vector spaces as matrices and vice-versa. Change of basis. Eigenvalues and eigenvectors.
Function spaces as linear spaces. Differentiation and integration as linear maps.

Groups

Groups of permutations. Groups of units of rings. Abelian and non-abelian groups. Cyclic groups.
Finite and infinite groups. Subgroups. Normal subgroups and quotients. Homomorphisms and isomorphisms. Homomorphic images and quotients.
Lagrange's Theorem. Order of an element. Groups of prime order and prime-square order. Sylow's Theorem as a partial converse of Lagrange's Theorem.

UNIT IV - ABSTRACT ANALYSIS

Topology

Metric as an abstractions of the absolute value of real and complex numbers. The Euclidean metric on R^n and C^n. The supremum metric and the integral metric on the set of real or complex valued functions on a closed interval.
Limit points in a metric space. Convergence of sequences in a metric space. Cauchy sequences. Complete metric spaces. Completion of a metric space. Cantor's Theorem and Baire's Theorem.
Topological spaces as generalizations of metric spaces. Non-metrizable spaces. Usual topology on R and C. Basis for a topology. Closure, interior and boundary of subsets.
Compactness and connectedness. Compact subsets and connected subsets of R and C. Heine-Borel Theorem for R^n
Convergence of sequences in a topological space. Non-uniqueness of limits. Hausdorff spaces. Continuity of functions between topological spaces. Preservation of compactness and connectedness. Non-continuity of inverse functions. Homeomorphisms.
Product topology and the topology of point-wise convergence in function spaces.

Functional Analysis

The norm on a vector space as a generalization of the length of a vector. Euclidean norm on R^n and C^n. Then ℓ^n_p and ℓ_p spaces. Supremum norm and integral norm on the space of continuous complex valued functions on a closed interval. The L^p spaces.
Closed and non-closed linear subspaces. Closure and interior of linear subspaces. Continuous linear maps between normed linear spaces. Non-continuity of the inverse. Boundedness and continuity.
Banach spaces. Open Mapping Theorem and the Bounded Inverse Theorem. Quotients as images for Banach spaces.
Inner products as generalization of the dot product. Examples of norms arising from inner products and not arising from any inner product. Parallelogram Law.
Orthogonality in inner product spaces. Orthogonal bases. Bessel's Inequality. Hilbert Spaces. Parseval's Identity. Fourier Expansion.
Continuous linear maps between Hilbert spaces. Adjoint of a linear map.